

TWO SHARP INEQUALITIES FOR LEHMER MEAN, IDENTRIC MEAN AND LOGARITHMIC MEAN

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Abstract. For $r \in \mathbb{R}$, the Lehmer mean of two positive numbers a and b is defined by

$$L_r(a, b) = \frac{a^{r+1} + b^{r+1}}{a^r + b^r}.$$

In this paper, we establish two sharp inequalities as follows: $I(a, b) > L_{-\frac{1}{3}}(a, b)$ and $L(a, b) > L_{-\frac{1}{3}}(a, b)$ for all $a, b > 0$ with $a \neq b$. Here $I(a, b) = \frac{1}{e} \left(\frac{b^b}{a^a} \right)^{\frac{1}{b-a}}$ and $L(a, b) = \frac{b-a}{\log b - \log a}$ denote the identric mean and logarithmic mean of two positive numbers a and b with $a \neq b$, respectively.

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