A PROOF OF A TRIGONOMETRIC INEQUALITY.
A GLIMPSE INSIDE THE MATHEMATICAL KITCHEN

JUAN ARIAS DE REYNA AND JAN VAN DE LUNE

Abstract. We prove the inequality

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{r^k \cos k\phi}{k+2} < \sum_{k=1}^{\infty} (-1)^{k+1} \frac{r^k}{k+2}$$

for $0 < r \leqslant 1$ and $0 < \phi < \pi$.

For the case $r = 1$ we give two proofs. The first one is by means of a general numerical technique (Maximal Slope Principle) for proving inequalities between elementary functions. The second proof is fully analytical. Finally we prove a general rearrangement theorem and apply it to the remaining case $0 < r < 1$.

Some of these inequalities are needed for obtaining general sharp bounds for the errors committed when applying the Riemann-Siegel expansion of Riemann's zeta function.

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REFERENCES

[1] J. ARIAS DE REYNA, *High Precision Computation of Riemann's Zeta Function by the Riemann-Siegel Formula, I*, Math. Comp., **80** (2011), 995–1009.
[2] H. M. EDWARDS, *Riemann's zeta function*, Reprint of the 1974 original [Academic Press, New York], Dover Publications Inc., Mineola, NY, 2001.
[3] M. FITZGERALD & I. JAMES, *The mind of the Mathematician*, The Johns Hopkins University Press, Baltimore, 2007.
[4] W. GABCKE, *Neue Herleitung und explizite Restabschätzung der Riemann-Siegel-Formel*, Dissertation, Göttingen, 1979.
[5] G. HARDY, *Divergent Series*, Oxford University Press, 1949.
[6] G. HARDY, J. E. LITTLEWOOD & G. PÓLYA, *Inequalities*, Second Ed., Cambridge University Press, Cambridge, 1991.
[7] J. VAN DE LUNE, *A note on the zeros of Flett's function*, Afdeling Zuivere Wiskunde, Report ZW 167. Mathematisch Centrum, Amsterdam, 1981.
http://repository.cwi.nl/search/fullrecord.php?publnr=6776
[8] J. VAN DE LUNE & H. H. J. TE RIELE, *Explicit computation of special zeros of partial sums of Riemann's zeta function*, Afdeling Zuivere Wiskunde, Report NW 44. Mathematisch Centrum, Amsterdam, 1977. http://repository.cwi.nl/search/fullrecord.php?publnr=9024

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