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A NOTE ON THE ISOPERIMETRIC INEQUALITY AND ITS STABILITY

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Abstract. In this paper, we deals with isoperimetric-type inequalities for closed convex curves in the Euclidean plane $\mathbb{R}^2$. We derive a family of parametric inequalities involving the following geometric functionals associated to a given convex curve with a simple Fourier series proof: length, area of the region included by the curve, area of the domain enclosed by the locus of curvature centers and integral of the radius of curvature. By using our isoperimetric-type inequalities, we also obtain some new geometric Bonnesen-type inequalities. Furthermore we investigate stability properties of such inequalities (near equality implies curve nearly circular).

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