

## OPTIMAL CONVEX COMBINATION BOUNDS OF SEIFFERT AND GEOMETRIC MEANS FOR THE ARITHMETIC MEAN

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*Abstract.* We find the greatest value  $\alpha$  and the least value  $\beta$  such that the double inequality  $\alpha T(a,b) + (1-\alpha)G(a,b) < A(a,b) < \beta T(a,b) + (1-\beta)G(a,b)$  holds for all  $a, b > 0$  with  $a \neq b$ . Here  $T(a,b)$ ,  $G(a,b)$ , and  $A(a,b)$  denote the Seiffert, geometric, and arithmetic means of two positive numbers  $a$  and  $b$ , respectively.

*Mathematics subject classification (2010):* 26E60.

*Keywords and phrases:* Seiffert mean, geometric mean, arithmetic mean.

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