OPTIMAL CONVEX COMBINATION BOUNDS OF SEIFFERT AND GEOMETRIC MEANS FOR THE ARITHMETIC MEAN

YU-MING CHU, CHENG ZONG AND GEN-DI WANG

Abstract. We find the greatest value α and the least value β such that the double inequality $\alpha T(a,b) + (1-\alpha)G(a,b) < A(a,b) < \beta T(a,b) + (1-\beta)G(a,b)$ holds for all $a,b > 0$ with $a \neq b$. Here $T(a,b)$, $G(a,b)$, and $A(a,b)$ denote the Seiffert, geometric, and arithmetic means of two positive numbers a and b, respectively.

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