

## $a(x)$ -MONOTONIC FUNCTIONS AND THEIR INEQUALITIES

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*Abstract.* In this paper  $a(x)$ -monotonic functions are defined and some inequalities for them are derived. Related analogues of the Lagrange and the Cauchy mean value theorems are also derived and means of the Cauchy type are generated. Furthermore, it is shown that the monotonicity of the Stolarsky means can be proved using the notion of generalized monotonic functions.

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