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FOUR INEQUALITIES OF VOLKMANN TYPE

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Abstract. We deal with four functional inequalities which are motivated by a result of A. Chaljub-Simon and P. Volkmann from 1994 and by several later results concerning the following two equations:

$$\max\{f(x+y), f(x-y)\} = f(x)+f(y) \quad \text{(for each } x,y\text{)}$$
$$\min\{f(x+y), f(x-y)\} = |f(x)-f(y)| \quad \text{(for each } x,y\text{)}.$$

The purpose of the paper is to establish some basic properties of the inequalities discussed and to compare them with some well known classical functional inequalities, such as the inequality of subadditivity or the inequality of Jensen-quasiconvexity.

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