

UNCERTAINTY PRINCIPLE IN TERMS OF ENTROPY FOR THE SPHERICAL MEAN OPERATOR

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Abstract. The logarithmic uncertainty principle in terms of entropy is proved for the spherical mean operator.

Mathematics subject classification (2010): Primary: 43A32, secondary: 42B10.

Keywords and phrases: Spherical mean operator, Hausdorff-Young theorem, Shanon entropy, logarithmic uncertainty principle.

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