

## UNCERTAINTY PRINCIPLE IN TERMS OF ENTROPY FOR THE SPHERICAL MEAN OPERATOR

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*Abstract.* The logarithmic uncertainty principle in terms of entropy is proved for the spherical mean operator.

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