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REFINED YOUNG INEQUALITY WITH KANTOROVICH CONSTANT

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Abstract. The Specht ratio $S(h)$ is the optimal constant in the reverse of the arithmetic-geometric mean inequality, i.e., if $0 < m \leqslant a,b \leqslant M$ and $h = \frac{M}{m}$, then $(1-\mu)a+\mu b \leqslant S(h)a^{1-\mu}b^{\mu}$ for all $\mu \in [0,1]$. Recently S. Furuichi proved that $(1-\mu)a+\mu b \geqslant S(h^r)a^{1-\mu}b^{\mu}$ for $a,b>0$, $\mu \in [0,1]$, where $h = \frac{b}{a}$ and $r = \min\{\mu, 1-\mu\}$. In this paper, we improve it by virtue of the Kantorovich constant, utilizing the refined scalar Young inequality we establish a weighted arithmetic-geometric-harmonic mean inequality for two positive operators. In the remainder of this work we focus on extending the refined weighted arithmetic-harmonic mean inequality to an operator version for another type of improvement.

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