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THE OPTIMAL POWER MEAN BOUNDS FOR TWO CONVEX COMBINATIONS OF $A-G-H$ MEANS

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Abstract. For $p \in \mathbb{R}$, let $M_p(a,b)$ denote the usual power mean of order p of positive real numbers a and b, and let $A = M_1$, $G = M_0$ and $H = M_{-1}$. We prove that the inequalities $M_0(a,b) \leqslant \frac{1}{3}[A(a,b)+G(a,b)+H(a,b)] \leqslant M_{\frac{\ln 2}{\ln 6}}(a,b)$ and $M_{-\frac{1}{6}}(a,b) \leqslant \frac{1}{2}[He(a,b)+H(a,b)] \leqslant M_{\frac{\ln 2}{\ln 6}}(a,b)$ hold for all positive real numbers a and b, with strict inequality for $a \neq b$, and that the orders of power means involved are optimal.

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