

RELATIVE CONVEXITY AND QUADRATURE RULES FOR THE RIEMANN–STIELTJES INTEGRAL

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Abstract. We develop Trapezoid, Midpoint, and Simpson's rules for the Riemann-Stieltjes integral, the latter two being new. These rules are completely natural when the notion of relative convexity is used.

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REFERENCES

- [1] S. D. CONTE & C. DE BOOR, *Elementary Numerical Analysis: An Algorithmic Approach*, 3rd edition, McGraw-Hill, 1980.
- [2] T. K. BOEHME, W. PREUSS & V. VAN DER WALL, *On a simple numerical method for computing Stieltjes integrals in reliability theory*, Prob. in Eng. & Inf. Sci. **5** (1991), 113–128.
- [3] K. DIETHELM, *A note on the midpoint rectangle formula for Riemann-Stieltjes integrals*, J. Stat. & Com. Simul. **74** (2004), 920–922.
- [4] S. S. DRAGOMIR, *Some inequalities of midpoint and trapezoid type for the Riemann-Stieltjes integral*, Nonlinear. Anal. **47** (2001), 2333–2340.
- [5] M. TORTORELLA, *Closed Newton-Cotes quadrature rules for Stieltjes integrals and numerical convolution of life distributions*, SIAM J. Sci. Stat. Comp. **11** (1990), 732–748.
- [6] M. TORTORELLA, *Numerical solutions of renewal-type integral equations*, INFORMS J. Comp. **17** (2005), 66–74.
- [7] M. XIE, W. PREUSS & L. R. CUI, *Error analysis of some integrations procedures for renewal equations and convolution integrals*, J. Stat. & Comp. Simul. **73** (2003), 59–70.
- [8] P. R. MERCER, *Hadamard's inequality and trapezoid rules for the Riemann-Stieltjes integral*, J. Math. Anal. Appl. **344** (2008), 921–926.
- [9] C. NICULESCU & L. E. PERSSON, *Convex Functions and their Applications*, CMS Books in Mathematics, Springer, 2006.