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THE ROLE OF CONCAVITY IN APPLICATIONS OF AVERY TYPE FIXED POINT THEOREMS TO HIGHER ORDER DIFFERENTIAL EQUATIONS

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Abstract. In this article we apply an extension of an Avery type fixed point theorem to a family of boundary value problems for higher order ordinary differential equations. The theorem employs concave and convex functionals defined on a cone in a Banach space. We begin by extending a known application to a right focal boundary value problem for a second order problem to a conjugate boundary value problem for a second order problem. We then extend inductively to a two point boundary value problem for a higher order equation. Concavity of differentiable functions plays a key role in the application to second order equations. A concept of generalized concavity plays the same key role in the application to the higher order equation.

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