SCHUR–CONVEXITY OF THE WEIGHTED ČEBYŠEV FUNCTIONAL II

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Abstract. In this paper the weighted Čebyšev functional $T(p; f, g; a, b)$ is regarded as a function of two variables

$$T(p; f, g; x, y) = \frac{\int_x^y p(t)f(t)g(t)dt}{\int_x^y p(t)dt} - \left(\frac{\int_x^y p(t)f(t)dt}{\int_x^y p(t)dt} \right) \left(\frac{\int_x^y p(t)g(t)dt}{\int_x^y p(t)dt} \right), \ (x, y) \in [a, b] \times [a, b]$$

where f, g and $p > 0$ are Lebesgue integrable functions. For a function

$$K(p; f, g; x, y) = \left(\int_x^y p(t)dt \right)^2 T(p; f, g; x, y) \ (x, y) \in [a, b] \times [a, b]$$

the property of Schur-covexity, Schur-geometric convexity, Schur-harmonic convexity and $(1, 1)$-convexity is proved.

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