Journal of Mathematical Inequalities
Volume 6, Number 2 (2012), 241–248
doi:10.7153/jmi-06-24

THE OPTIMAL CONVEX COMBINATION BOUNDS OF ARITHMETIC AND HARMONIC MEANS IN TERMS OF POWER MEAN

WEI-FENG XIA, WALTHER JANOUS AND YU-MING CHU

Abstract. In this paper, we answer the question: What are the greatest value $p = p(\alpha)$ and least value $q = q(\alpha)$ such that the double inequality $M_p(a,b) \leqslant \alpha A(a,b) + (1-\alpha)H(a,b) \leqslant M_q(a,b)$ holds for any $\alpha \in (0,1)$ and all $a,b > 0$? Here, $M_p(a,b)$, $A(a,b)$, and $H(a,b)$ are the p-th power, arithmetic, and harmonic means of a and b, respectively

Mathematics subject classification (2010): 26E60, 26D15.
Keywords and phrases: Power mean, arithmetic mean, harmonic mean.

REFERENCES

[1] S. H. WU, *Generalization and sharpness of the power means inequality and their applications*, J. Math. Anal. Appl., **312**(2): 637–652 (2005).
[2] K. C. RICHARDS, *Sharp power mean bounds for the Gaussian hypergeometric function*, J. Math. Anal. Appl., **308**(1): 303–313 (2005).
[3] W. L. WANG, J. J. WEN, H. N. SHI, *Optimal inequalities involving power means*, *Acta Math. Sinica* (in Chinese), **47**(6): 1053–1062 (2004).
[4] P. A. HÄSTÖ, *Optimal inequalities between Seiffert's mean and power means*, Math. Inequal. Appl., **7**(1): 47–53 (2004).
[5] H. ALZER, S. L. QIU,, *Inequalities for means in two variables*, Arch. Math. (Basel), **80**(2): 201–215 (2003).
[6] H. ALZER, *A power mean inequality for the gamma function*, Monatsh. Math., **131**(3): 179–188 (2000).
[7] C. D. TARNAVAS, D. D. TARNAVAS, *An inequality for mixed power means*, Math. Inequal. Appl., **2**(2): 175–181 (1999).
[8] H. ALZER, *Inequalities for the gamma function*, Proc. Amer. Math. Soc., **128**(1): 141–147 (2000)
[9] H. T. KU, M. C. KU, X. M. ZHANG, *Generalized power means and interpolating inequalities*, Proc. Amer. Math. Soc., **127**(1): 145–154 (1999).
[10] B. MOND, J. E. PEČARIĆ, *Difference and ratio operator inequalities in Hilbert space*, Houston J. Math., **21**(1): 103–108 (1995).
[11] B. MOND, J. E. PEČARIĆ, *Generalized power means for matrix functions*, Publ. Math. Debrecen, **46**(1–2): 33–39 (1995).
[12] J. E. PEČARIĆ, *Generalization of the power means and their inequalities*, J. Math. Anal. Appl., **161**(2): 395–404 (1991).
[13] H. S. WILF, *Some examples of combinatorial averaging*, Amer. Math. Monthly, **92**(4): 250–261 (1985).
[14] C. O. IMORU, *The power mean and the logarithmic mean*, Internat. J. Math. Math. Sci., **5**(2): 337–343 (1982).
[15] K. B. STOLARSKY, *The power and generalized logarithmic means*, Amer. Math. Monthly, **87**(7): 545–548 (1980).
[16] K. V. BHAGWAT, R. SUBRAMANIAN, *Inequalities between means of positive operators*, Math. Proc. Cambridge Philos. Soc., **83**(3): 393–401 (1978).
[17] T. P. LIN, *The power mean and the logarithmic mean*, Amer. Math. Monthly, **81**: 879–883 (1974).
[18] H. ALZER, W. JANOUS, *Solution of problem 8**, Crux.Math., **13**: 173–178 (1987).

Paper JMI-06-24

[19] P. S. BULLEN, D. S. MITRINOVIĆ, P. M. VASIĆ, *Means and Their Inequalities*, Dordrecht: D. Reidel Publishing Co., 1988.
[20] W. JANOUS, *A note on generalized Heronian means*, Math. Inequal. Appl., **4**(3): 369–375 (2001).