Journal of
Mathematical
Inequalities
Volume 6, Number 2 (2012), 253–271 doi:10.7153/jmi-06-26

ON POWER SUMS OF CONVEX FUNCTIONS IN LOCAL MINIMUM ENERGY PROBLEM

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Abstract. In this paper, new inequalities on the power sums of a convex function are derived and the monotonically decreasing nature of the Riemann sum of a function including a certain strong convexity is shown. It is also shown that a derived inequality has a direct implication in a local minimum energy problem in two-dimensional hexagonal packing.

Mathematics subject classification (2010): 26D15, 26A51, 74G65.
Keywords and phrases: Convex functions, strong convexity, power sums, Riemann sum, monotonicity, energy, triangular lattice.

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