

A NEW NONLINEAR INTEGRAL INEQUALITY OF WENDROFF TYPE WITH CONTINUOUS AND WEAKLY SINGULAR KERNEL AND ITS APPLICATION

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Abstract. The main objective of this paper is to establish some new explicit bounds for nonlinear integral inequalities of Wendroff type with continuous and weakly singular kernel, which generalized some known inequalities for functions in two variables and can be furnished a handy tool for the study of qualitative as well as quantitative properties of solutions of nonlinear differential equations. Some applications are also given to illustrate the usefulness of our results.

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