

WEIGHTED OSTROWSKI, TRAPEZOID AND GRÜSS TYPE INEQUALITIES ON TIME SCALES

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Abstract. In this paper we first derive a weighted Montgomery identity on time scales and then establish weighted Ostrowski, trapezoid and Grüss type inequalities on time scales, respectively. These results not only provide a generalization of the known results, but also give some other interesting inequalities on time scales as special cases.

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REFERENCES

- [1] R. AGARWAL, M. BOHNER AND A. PETERSON, *Inequalities on time scales: a survey*, Math. Inequal. Appl. **4**, 4 (2001), 535–557.
- [2] F. M. ATICI, D. C. BILES AND A. LEBEDINSKY, *An application of time scales to economics*, Math. Comput. Modelling **43**, 7–8 (2006), 718–726.
- [3] M. BOHNER, M. FAN AND J. M. ZHANG, *Periodicity of scalar dynamic equations and applications to population models*, J. Math. Anal. Appl. **330** (2007), 1–9.
- [4] M. BOHNER AND A. PETERSON, *Dynamic equations on time scales*, Birkhäuser Boston, Boston, MA, 2001.
- [5] M. BOHNER AND A. PETERSON, *Advances in dynamic equations on time scales*, Birkhäuser Boston, Boston, MA, 2003.
- [6] M. BOHNER AND T. MATTHEWS, *The Grüss inequality on time scales*, Commun. Math. Anal. **3**, 1 (2007), 1–8 (electronic).
- [7] M. BOHNER AND T. MATTHEWS, *Ostrowski inequalities on time scales*, JIPAM. J. Inequal. Pure Appl. Math. **9**, 1 (2008), Article 6, 8 pp.
- [8] M. BOHNER, T. MATTHEWS AND A. TUNA, *Diamond- α Grüss type inequalities on time scales*, Int. J. Dyn. Syst. Differ. Equ. **3**, 1/2 (2011), 234–247.
- [9] P. CERONE, S. S. DRAGOMIR AND J. ROUMELIOTIS, *An inequality of Ostrowski type for mappings whose second derivatives are bounded and applications*, RGMIA Research Report Collection **1** (1998), 33–39.
- [10] P. CERONE AND S. S. DRAGOMIR, *Midpoint-type rules from an inequalities point of view*, in Handbook of analytic-computational methods in applied mathematics, 135–200, Chapman & Hall/CRC, Boca Raton, FL.
- [11] C. DINU, *Ostrowski type inequalities on time scales*, An. Univ. Craiova Ser. Mat. Inform. **34** (2007), 43–58.
- [12] S. HILGER, *Ein Maßkettenkalkül mit Anwendung auf Zentrumsmannigfaltigkeiten*, Ph.D. thesis, Universität Würzburg, Würzburg, Germany, 1988.
- [13] B. KARPUZ AND U. M. ÖZKAN, *Generalized Ostrowski's inequality on time scales*, JIPAM. J. Inequal. Pure Appl. Math. **9**, 4 (2008), Article 112, 7 pp.
- [14] V. LAKSHMIKANTHAM, S. SIVASUNDARAM AND B. KAYMAKÇALAN, *Dynamic systems on measure chains*, Kluwer Acad. Publ., Dordrecht, 1996.
- [15] G. LAPENTA, *Particle simulations of space weather*, J. Comput. Phys. **231**, 3 (2012), 795–821.

- [16] W. N. LI, *Some delay integral inequalities on time scales*, *Comput. Math. Appl.* **59**, 6 (2010), 1929–1936.
- [17] W. N. LI AND W. H. SHENG, *Some Gronwall type inequalities on time scales*, *J. Math. Inequal.* **4**, 1 (2010), 67–76.
- [18] W. J. LIU, *Some weighted integral inequalities with a parameter and applications*, *Acta Appl. Math.* **109**, 2 (2010), 389–400.
- [19] W. J. LIU AND Q.-A. NGÔ, *An Ostrowski-Grüss type inequality on time scales*, *Comput. Math. Appl.* **58**, 6 (2009), 1207–1210.
- [20] W. J. LIU AND Q.-A. NGÔ, *A generalization of Ostrowski inequality on time scales for k points*, *Appl. Math. Comput.* **203**, 2 (2008), 754–760.
- [21] W. J. LIU, Q.-A. NGÔ AND W. B. CHEN, *A perturbed Ostrowski-type inequality on time scales for k points for functions whose second derivatives are bounded*, *J. Inequal. Appl.* **2008**, Art. ID 597241, 12 pp.
- [22] W. J. LIU, Q.-A. NGÔ AND W. B. CHEN, *Ostrowski type inequalities on time scales for double integrals*, *Acta Appl. Math.* **110**, 1 (2010), 477–497.
- [23] W. J. LIU, Q.-A. NGÔ AND W. B. CHEN, *A new generalization of Ostrowski type inequality on time scales*, *An. Ştiinţ. Univ. “Ovidius” Constanţa Ser. Mat.* **17**, 2 (2009), 101–114.
- [24] D. S. MITRINOVIĆ, J. E. PEČARIĆ AND A. M. FINK, *Inequalities involving functions and their integrals and derivatives*, Kluwer Acad. Publ., Dordrecht, 1991.
- [25] Q.-A. NGÔ AND W. J. LIU, *A sharp Grüss type inequality on time scales and application to the sharp Ostrowski-Grüss inequality*, *Commun. Math. Anal.* **6**, 2 (2009), 33–41.
- [26] A. OSTROWSKI, *Über die Absolutabweichung einer differentiiierbaren Funktion von ihrem Integralmittelwert*, *Comment. Math. Helv.* **10**, 1 (1937), 226–227.
- [27] B. G. PACHPATTE, *On trapezoid and Grüss-like integral inequalities*, *Tamkang J. Math.* **34**, 4 (2003), 365–369.
- [28] M. Z. SARIKAYA, *A note on Grüss type inequalities on time scales*, *Dynam. Systems Appl.* **17**, 3–4 (2008), 663–666.
- [29] M. Z. SARIKAYA, N. AKTAN AND H. YILDIRIM, *On weighted Čebyšev-Grüss type inequalities on time scales*, *J. Math. Inequal.* **2**, 2 (2008), 185–195.
- [30] M. R. SIDI AMMI AND D. F. M. TORRES, *Combined dynamic Grüss inequalities on time scales*, *Journal of Mathematical Sciences* **161**, 6 (2009), 792–802.
- [31] C. SORIA-HOYO, F. PONTIGA AND A. CASTELLANOS, *A PIC based procedure for the integration of multiple time scale problems in gas discharge physics*, *J. Comput. Phys.* **228** (2009), 1017–1029.
- [32] H. M. SRIVASTAVA, K.-L. TSENG, S.-J. TSENG AND J.-C. LO, *Some weighted Opial-type inequalities on time scales*, *Taiwanese J. Math.* **14**, 1 (2010), 107–122.
- [33] K.-L. TSENG, G.-S. YANG AND S. S. DRAGOMIR, *Generalizations of weighted trapezoidal inequality for mappings of bounded variation and their applications*, *Math. Comput. Modelling* **40**, 1–2 (2004), 77–84.
- [34] K.-L. TSENG, S. R. HWANG AND S. S. DRAGOMIR, *Generalizations of weighted Ostrowski type inequalities for mappings of bounded variation and their applications*, *Comput. Math. Appl.* **55**, 8 (2008), 1785–1793.
- [35] A. TUNA AND D. DAGHAN, *Generalization of Ostrowski and Ostrowski-Grüss type inequalities on time scales*, *Comput. Math. Appl.* **60**, 3 (2010), 803–811.
- [36] A. TUNA, Y. JIANG AND W. J. LIU, *WEIGHTED OSTROWSKI, Ostrowski-Grüss and Ostrowski-Čebyšev Type Inequalities on Time Scales*, *Publ. Math. Debrecen* **80** (2012), to appear.
- [37] S. F. WANG, Q. L. XUE, W. J. LIU, *Further generalization of Ostrowski-Grüss type inequalities*, *Adv. Appl. Math. Anal.* **3**, 1 (2008), 17–20.
- [38] C.-C. YEH, *Ostrowski’s inequality on time scales*, *Appl. Math. Lett.* **21**, 4 (2008), 404–409.