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ON A CONJECTURE OF SCHINZEL AND ZASSENHAUS

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Abstract. A. Schinzel and H. Zassenhaus had the following conjecture regarding algebraic integers: If $\alpha \neq 0$ is an algebraic integer of degree n which is not a root of unity, then there exists a constant $c > 0$ such that

$$|\overline{\alpha}| \geqslant 1 + \frac{c}{n},$$

where $|\overline{\alpha}| = \max_{1 \leqslant i \leqslant n} |\alpha_i|$, $\alpha_1 = \alpha$ and $\alpha_2, \cdots, \alpha_n$ are the conjugates of α.

We give some partial solutions to this conjecture in this paper via spectral properties.

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