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APPROXIMATE FUNCTIONAL INEQUALITIES BY ADDITIVE MAPPINGS

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Abstract. Let n be a given positive integer, G an n-divisible abelian group, X a normed space and $f: G \to X$. We prove a generalized Hyers-Ulam stability of the following functional inequality

$$\|f(x)+f(y)+nf(z)\| \leqslant \left\|nf\left(\frac{x+y}{n}+z\right)\right\| + \varphi(x,y,z), \quad \forall x,y,z \in G,$$

which has been introduced in [3], under some conditions on $\varphi : G\times G\times G \to [0,\infty)$.

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