

REFINEMENTS AND SHARPNESS OF SOME NEW HUYGENS TYPE INEQUALITIES

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Abstract. In the article, some Huygens inequalities involving trigonometric and hyperbolic functions are refined and sharpened.

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