Journal of Mathematical Inequalities
Volume 6, Number 3 (2012), 493–500
doi:10.7153/jmi-06-46

REFINEMENTS AND SHARPNESS OF SOME NEW HUYGENS TYPE INEQUALITIES

YUN HUA

Abstract. In the article, some Huygens inequalities involving trigonometric and hyperbolic functions are refined and sharpened.

Mathematics subject classification (2010): Primary 26D05; Secondary 33B10.
Keywords and phrases: Refinement, sharpening, Huygens inequality, Wilker inequality, trigonometric function, hyperbolic function.

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