

LOGARITHMIC CONVEXITY OF GINI MEANS

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Abstract. In the paper, alternative proofs for the monotonicity and logarithmic convexity of Gini means and related functions are presented by using new approaches and techniques.

Mathematics subject classification (2010): Primary 26A48, 26A51; Secondary 26B25, 26D07.

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