

## NEW SHARP BOUNDS FOR IDENTRIC MEAN IN TERMS OF LOGARITHMIC MEAN AND ARITHMETIC MEAN

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*Abstract.* Let  $x, y > 0$  with  $x \neq y$ . We give new sharp bounds for identric mean  $I = e^{-1} (x^x / y^y)^{1/(x-y)}$  in terms of logarithmic mean  $L = (x - y) / (\ln x - \ln y)$  and arithmetic mean  $A = (x + y) / 2$ :

$$\left(\frac{1}{2}L^{p_0} + \frac{1}{2}A^{p_0}\right)^{1/p_0} < I < \left(\frac{1}{2}L^{\bar{p}_0} + \frac{1}{2}A^{\bar{p}_0}\right)^{1/\bar{p}_0},$$

where  $p_0 = 8/5$  and  $\bar{p}_0 = (\ln 2) / (1 - \ln 2)$  are the best possible constants.

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