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NEW SHARP BOUNDS FOR IDENTRIC MEAN IN TERMS OF LOGARITHMIC MEAN AND ARITHMETIC MEAN

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Abstract. Let $x,y>0$ with $x\neq y$. We give new sharp bounds for identric mean $I=e^{-1}\left(x^x/y^y\right)^{1/(x-y)}$ in terms of logarithmic mean $L=(x-y)/(\ln x-\ln y)$ and arithmetic mean $A=(x+y)/2$:

$$\left(\tfrac{1}{2}L^{p_0}+\tfrac{1}{2}A^{p_0}\right)^{1/p_0}<I<\left(\tfrac{1}{2}L^{\tilde{p}_0}+\tfrac{1}{2}A^{\tilde{p}_0}\right)^{1/\tilde{p}_0},$$

where $p_0=8/5$ and $\tilde{p}_0=(\ln 2)/(1-\ln 2)$ are the best possible constants.

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REFERENCES

[1] G. D. ANDERSON, M. K. VAMANAMURTHY AND M. VUORINEN, *Conformal Invariants, Inequalities, and Quasiconformal Maps*, New York 1997.
[2] H. ALZER, *Aufgabe 987*, Elem. Math., **43**, 93 (1988), (German).
[3] H. ALZER AND S.-L. QIU, *Inequalities for means in two variables*, Arch. Math. (Basel), **80** (2003), 201–215.
[4] M. BIERNACKI AND J. KRZYZ, *On the monotonity of certain functionals in the theory of analytic functions*, Annales Universitatis Mariae Curie-Sklodowska, **9** (1995), 135–147.
[5] P. S. BULLEN, D. S. MITRINOVIĆ AND P. M. VASIĆ, *Means and Their Inequalities*, Dordrecht 1988.
[6] O. KOUBA, *New bounds for the identric mean of two arguments*, J. Inequal. Pure Appl. Math., **9**, 3 (2008), Art. 71, 6 pages.
[7] E. NEUMAN AND J. SÁNDOR, *On certain means of two arguments and their extensions*, Int. J. Math. Math. Sci., **2003**, 16(2003), 981–993.
[8] E. NEUMAN AND J. SÁNDOR, *Inequalities involving Stolarsky and Gini means*, Math. Pannon., **14**, 1 (2003), 29-44.
[9] A. O. PITTINGER, *Inequalities between arithmetic and logarithmic means*, Univ. Beograd Publ. Elektr. Fak. Ser. Mat. Fiz, **680** (1980), 15–18.
[10] Y.-F. QIU, M.-K. WANG, Y.-M. CHU AND G.-D. WANG, *Two sharp inequalities for Lehmer mean, identric mean and logarithmic mean*, J. Math. Inequal., **5**, 3 (2011), 301–306.
[11] J. SÁNDOR, *On the identric and logarithmic means*, Aequat. Math., **40** (1990), 261–270 .
[12] J. SÁNDOR, *A note on some inequalities for means*, Arch. Math., **56** (1991), 471–473.
[13] J. SÁNDOR, *Two inequalities for means*, Int. J. Math. Math. Sci., **18**, 3 (1995), 621–623. MR 96b:26030. Zbl 827.26016.
[14] J. SÁNDOR, *On certain inequalities for means III*, Arch. Math., **76** (2001), 34–40.
[15] J. SÁNDOR AND T. TRIF, *Some new inequalities for means of two arguments*, Internat. J. Math. Math. Sci., **25** (2001), 525–532.
[16] K. B. STOLARSKY, *Generalizations of the logarithmic mean*, Math. Mag., **48** (1975), 87–92.
[17] K. B. STOLARSKY, *The power and generalized logarithmic means*, Amer. Math. Monthly, **87** (1980), 545–548.
[18] T. TRIF, *Note on certain inequalities for means in two variables*, J. Inequal. Pure Appl. Math., **6**, 2(2005), Art. 43; available online at `http://jipam.vu.edu.au/article.php?sid=512`.

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[19] M. K. VAMANAMURTHY AND M. VUORINEN, *Inequalities for means*, J. Math. Anal. Appl., **183** (1994), 155–166.
[20] ZH.-H. YANG, *Exponential mean and logarithmic mean*, Mathematics in Practice and Theory, **1987**, 4 (1987), 76–78, (Chinese)
[21] ZH.-H. YANG, *On the homogeneous functions with two parameters and its monotonicity*, J. Inequal. Pure Appl. Math., **6**, 4 (2005), Art. 101; available online at http://jipam.vu.edu.au/images/155_05_JIPAM/155_05.pdf.
[22] ZH.-H. YANG, *On the log-convexity of two-parameter homogeneous functions*, Math. Inequal. Appl., **10**, 3 (2007), 499–516.
[23] ZH.-H. YANG, *On the monotonicity and log-convexity of a four-parameter homogeneous mean*, J. Inequal. Appl., **2008** (2008), Art. ID 149286, 12 pages, doi:10.1155/2008/149286; available online at http://www.hindawi.com/GetArticle.aspx?doi=10.1155/2008/149286.
[24] ZH.-H. YANG, *Log-convexity of ratio of the two-parameter symmetric homogeneous functions and an application*, J. Inequal. Spec. Func., **1**, 1(2010), 16-29; available online athttp://www.ilirias.com.
[25] ZH.-H. YANG, *The log-convexity of another class of one-parameter means and its applications*, Bull. Korean Math. Soc., **49**, 1(2012), 33–47; available online at http://dx.doi.org/10.4134/BKMS.2012.49.1.033.
[26] L. ZHU, *New inequalities for means in two variables*, Math. Inequal. Appl., **11**, 2 (2008), 229–235.