Journal of Mathematical Inequalities
Volume 6, Number 4 (2012), 545–555
doi:10.7153/jmi-06-52

SHARP INEQUALITIES RELATED TO ONE–PARAMETER MEAN AND GINI MEAN

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Abstract. In the present paper, we answer the question: For $\alpha+\beta\in(0,1)$, what are the greatest values p,s_1 and the least values q,s_2 such that the inequalities

$$J_p(a,b)\leqslant A^{\alpha}(a,b)G^{\beta}(a,b)H^{1-\alpha-\beta}(a,b)\leqslant J_q(a,b)$$

and

$$G_{s_1,1}(a,b)\leqslant A^{\alpha}(a,b)G^{\beta}(a,b)H^{1-\alpha-\beta}(a,b)\leqslant G_{s_2,1}(a,b)$$

hold for all $a,b>0$ with $a\neq b$? where $J_p(a,b)$, $A(a,b)$, $G(a,b)$, $H(a,b)$ and $G_{s,1}(a,b)$ are the one-parameter mean, arithmetic mean, geometric mean, harmonic mean and Gini mean for two positive numbers a and b, respectively.

Mathematics subject classification (2010): 26D99.

Keywords and phrases: One-parameter mean, Gini mean, arithmetic mean, geometric mean, harmonic mean.

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Paper JMI-06-52