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SHARP BOUNDS FOR THE NEUMAN–SÁNDOR MEAN IN TERMS OF GENERALIZED LOGARITHMIC MEAN

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Abstract. In this paper, we find the largest value α and least value β such that the double inequality $L_\alpha(a,b) < M(a,b) < L_\beta(a,b)$ holds for all $a,b > 0$ with $a \neq b$. Here, $M(a,b)$ and $L_p(a,b)$ are the Neuman-Sándor and p-th generalized logarithmic means of a and b, respectively.

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REFERENCES

[1] K. B. STOLARSKY, *Generalizations of the logarithmic mean*, Math. Mag. **48** (1975), 87–92.
[2] E. NEUMAN AND J. SÁNDOR, *On the Schwab-Borchardt mean*, Math. Pannon. **14**, 2 (2003), 253–266.
[3] M. K. WANG, Z. K. WANG AND Y. M. CHU, *An optimal double inequality between geometric and identric means*, Appl. Math. Lett. **25**, 3 (2012), 471–475.
[4] Y. M. CHU, M. K. WANG AND S. L. QIU, *Optimal combinations bounds of root-square and arithmetic means for Toader mean*, Proc. Indian Acad. Sci. Math. Sci. **122**, 1 (2012), 41–51.
[5] Y. M. CHU, M. K. WANG AND Z. K. WANG, *An optimal double inequality between Seiffert and geometric means*, J. Appl. Math. **2011**, Article ID 261237, 6 pages.
[6] Y. M. CHU, S. W. HOU AND W. M. GONG, *Inequalities between logarithmic, harmonic, arithmetic and centroidal means*, J. Math. Anal. **2**, 2 (2011), 1–5.
[7] Y. M. CHU, M. K. WANG AND Z. K. WANG, *A best-possible double inequality between Seiffert and harmonic means*, J. Inequal. Appl., **2011** (2011): 94.
[8] Y. M. CHU AND M. K. WANG, *Optimal inequalities between harmonic, geometric, logarithmic, and arithmetic-geometric means*, J. Appl. Math. **2011**, Article ID 618929, 9 pages.
[9] Y. M. CHU, M. K. WANG AND Z. K. WANG, *A sharp double inequality between harmonic and identric means*, Abstr. Appl. Anal. **2011**, Article ID 657935, 7 pages.
[10] Y. M. CHU, M. K. WANG AND W. M. GONG, *Two sharp double inequalities for Seiffert mean*, J. Inequal. Appl. **2011** (2011), 44.
[11] Y. M. CHU, S. S. WANG AND C. ZONG, *Optimal lower power mean bound for the convex combination of harmonic and logarithmic means*, Abstr. Appl. Anal. **2011**, Article ID 520648, 9 pages.
[12] Y. M. CHU, C. ZONG AND G. D. WANG, *Optimal convex combination bounds of Seiffert and geometric means for the arithmetic mean*, J. Math. Inequal. **5**, 3 (2011), 429–434.
[13] Y. M. CHU AND B. Y. LONG, *Sharp inequalities between means*, Math. Inequal. Appl. **14**, 3 (2011), 647–655.
[14] Y. F. QIU, M. K. WANG, Y. M. CHU AND G. D. WANG, *Two sharp inequalities for Lehmer mean, identric mean and logarithmic mean*, J. Math. Inequal. **5**, 3 (2011), 301–306.
[15] M. Y. SHI, Y. M. CHU AND Y. P. JIANG, *Optimal inequalities related to the power, harmonic and identric means*, Acta Math. Sci. **31A**, 5 (2011), 1377–1384.
[16] Y. M. CHU AND W. F. XIA, *Two optimal double inequalities between power mean and logarithmic mean*, Comput. Math. Appl. **60**, 1 (2010), 83–89.

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[17] W. F. XIA, Y. M. CHU AND G. D. WANG, *The optimal upper and lower power mean bounds for a convex combination of the arithmetic and logarithmic means*, Abstr. Appl. Anal. **2010**, Article ID 604804, 9 pages.
[18] M. K. WANG, Y. M. CHU AND Y. F. QIU, *Some comparison inequalities for generalized Muirhead and identric means*, J. Inequal. Appl. **2010**, Article ID 295620, 10 pages.
[19] M. Y. SHI, Y. M. CHU AND Y. P. JIANG, *Three best inequalities for means in two variables*, Int. Math. Forum **5**, 22 (2010), 1059–1066.
[20] W. F. XIA AND Y. M. CHU, *Optimal inequalities related to the logarithmic, identric, arithmetic and harmonic means*, Rev. Anal. Numér. Théor. Approx. **39**, 2 (2010), 176–183.
[21] Y. M. CHU AND W. F. XIA, *Inequalities for generalized logarithmic means*, J. Inequal. Appl. **2009**, Article ID 763252, 7 pages.
[22] M. Y. SHI, Y. M. CHU AND Y. P. JANG, *Optimal inequalities among various means of two arguments*, Abstr. Appl. Anal. **2009**, Article ID 694394, 10 pages.
[23] H. N. SHI AND S. H. WU, *Refinement of an inequality for the generalized logarithmic mean*, Chinese Quart. J. Math. **23**, 4 (2008), 594–599.
[24] F. QI, S. X. CHEN AND C. P. CHEN, *Monotonicity of ratio between the generalized logarithmic means*, Math. Inequal. Appl. **10**, 3 (2007), 559–564.
[25] X. LI, C. P. CHEN AND F. QI, *Monotonicity result for generalized logarithmic means*, Tamkang J. Math. **38**, 2 (2007), 177–181.
[26] C. P. CHEN AND F. QI, *Monotonicity properties for generalized logarithmic means*, Aust. J. Math. Anal. Appl. **1**, 2 (2004), Article 2, 4 pages.
[27] H. ALZER AND S. L. QIU, *Inequalities for means in two variables*, Arch. Math. **80**, 2 (2003), 201–215.
[28] B. MOND, C. E. M. PEARCE AND J. PEČARIĆ, *The logarithmic mean is a mean*, Math. Commun. **2**, 1 (1997), 35–39.
[29] P. KAHLIG AND J. MATKOWSKI, *Functional equations involving the logarithmic mean*, Z. Angew. Math. Mech. **76**, 7 (1996), 385–390.
[30] C. E. M. PEARCE AND J. PEČARIĆ, *Some theorems of Jensen type for generalized logarithmic means*, Rev. Roumaine Math. Pures Appl. **40**, 9/10 (1995), 789–795.
[31] A. O. PITTENGER, *The logarithmic mean in n variables*, Amer. Math. Monthly **92**, 2 (1985), 99–104.
[32] K. B. STOLARSKY, *The power and generalized logarithmic means*, Amer. Math. Monthly **87**, 7 (1980), 545–548.
[33] T. P. LIN, *The power mean and the logarithmic mean*, Amer. Math. Monthly **81** (1974), 879–883.
[34] P. S. BULLEN, D. S. MITRINOVIĆ AND P. M. VASIĆ, *Means and Their Inequalities*, D. Reidel publishing Co., Dordrecht, 1988.
[35] A. O. PITTENGER, *Inequalities between arithmetic and logarithmic means*, Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat. Fiz. **678–715** (1980), 15–18.
[36] B. Y. LONG AND Y. M. CHU, *Optimal inequalities for generalized logarithmic, arithmetic, and geometric means*, J. Inequal. Appl. **2010**, Article ID 806825, 10 pages.
[37] Y. M. CHU AND B. Y. LONG, *Best possible inequalities between generalized logarithmic mean and classical means*, Abstr. Appl. Anal. **2010**, Article ID 303286, 13 pages.
[38] E. NEUMAN AND J. SÁNDOR, *On the Schwab-Borchardt mean* II, Math. Pannon. **17**, 1 (2006), 49–59.