

SHARP BOUNDS FOR THE NEUMAN-SÁNDOR MEAN IN TERMS OF GENERALIZED LOGARITHMIC MEAN

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Abstract. In this paper, we find the largest value α and least value β such that the double inequality $L_\alpha(a, b) < M(a, b) < L_\beta(a, b)$ holds for all $a, b > 0$ with $a \neq b$. Here, $M(a, b)$ and $L_p(a, b)$ are the Neuman-Sándor and p -th generalized logarithmic means of a and b , respectively.

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