

## ON MONOTONICITY OF SOME OPERATOR FUNCTIONS RELATED TO ORDER PRESERVING OPERATOR INEQUALITIES

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*Abstract.* In this paper, we shall show an equivalence relation between extensions of order preserving operator inequalities and monotonicity of related operator functions.

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