

SOME SHARP INEQUALITIES INVOLVING RECIPROCAL OF THE SEIFFERT AND OTHER MEANS

WEI-DONG JIANG

Abstract. In the paper, by establishing the monotonicity of some functions involving the sine and cosine functions, we find some new sharp inequalities involving the reciprocals of the Seiffert, contra-harmonic, arithmetic, geometric and root-square means of two positive real numbers a and b with $a \neq b$.

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