PERTURBATION BOUNDS FOR SINGULAR VALUES OF MATRICES WITH SINGLETONS

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Abstract. This paper considers bounds for the singular values of a matrix with a row- or column-singleton if such an element is perturbed. A multiplicative approach to such perturbations may lead to sharper bounds than the usual additive Weyl bounds. These bounds immediately give the interlace property for the singular values. These results are used for perturbation bounds if a tridiagonal matrix with a zero diagonal is perturbed in the last pair of off-diagonal elements. Such perturbation bounds are then applied to bound perturbations of the eigenvalues and the first component of the eigenvectors, which has a direct application in the computation of the Gaussian quadrature formulae for compression splines.

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