Journal of Mathematical Inequalities
Volume 6, Number 4 (2012), 615–623
doi:10.7153/jmi-06-59

SOME INEQUALITIES FOR UNITARILY INVARIANT NORMS

XINGKAI HU

Abstract. This paper aims to present some inequalities for unitarily invariant norms. In section 2, we give a refinement of the Cauchy-Schwarz inequality for matrices. In section 3, we obtain an improvement for the result of Bhatia and Kittaneh [Linear Algebra Appl. 308 (2000) 203-211]. In section 4, we establish an improved Heinz inequality for the Hilbert-Schmidt norm. Finally, we present an inequality involving positive definite matrix and Hilbert-Schmidt norm. Then we use it to discuss the conjecture on the Hilbert-Schmidt norm of matrices proposed by Sloane and Harwit and the conjecture is proved for some special matrices.

Mathematics subject classification (2010): 15A18; 15A42; 15A60.
Keywords and phrases: Unitarily invariant norms, positive semidefinite matrices, Cauchy-Schwarz inequality, convex function, Heinz inequality.

REFERENCES

[1] S. WANG, L. ZOU, Y. JIANG, *Some inequalities for unitarily invariant norms of matrices*, J. Inequal. Appl. 2011, 2011:10.
[2] R. A. HORN, R. MATHIAS, *Cauchy–Schwarz inequalities associated with positive semidefinite matrices*, Linear Algebra Appl. **142** (1990) 63–82.
[3] R. A. HORN, R. MATHIAS, *An analog of the Cauchy–Schwarz inequality for Hadamard products and unitarily invariant norms*, SIAM J. Matrix Anal. Appl. **11** (1990) 481–498.
[4] R. BHATIA, C. DAVIS, *A Cauchy–Schwarz inequality for operators with applications*, Linear Algebra Appl. **223/224** (1995) 119–129.
[5] R. BHATIA, *Matrix Analysis*, Springer-Verlag, 1997.
[6] F. HIAI, X. ZHAN, *Inequalities involving unitarily invariant norms and operator monotone functions*, Linear Algebra Appl. **341** (2002) 151–169.
[7] R. BHATIA, F. KITTANEH, *Notes on matrix arithmetic–geometric mean inequalities*, Linear Algebra Appl. **308** (2000) 203–211.
[8] F. KITTANEH, *Norm inequalities for fractional powers of positive operators*, Lett. Math. Phys. **27** (1993) 279–285.
[9] R. BHATIA, C. DAVIS, *More matrix forms of the arithmetic-geometric mean inequality*, SIAM J. Matrix Anal. Appl. **14** (1993) 132–136.
[10] F. KITTANEH, Y. MANASRAH, *Improved Young and Heinz inequalities for matrices*, J. Math. Anal. Appl. **361** (2010) 262–269.
[11] X. ZHAN, *Matrix theory*, Beijing: Higher Education Press, 2008.
[12] N. J. A. SLOANE, M. HARWIT, *Masks for Hadamard transform optics, and weighing designs*, Appl. Optics. **15** (1976) 107–114.
[13] L. ZOU, Y. JIANG, X. HU, *A note on a conjecture on the Frobenius norm of matrices*, Journal of Shandong University (Natural Science), **45** (4) (2010) 48–50.
[14] L. ZOU, *On a conjecture concerning the Frobenius norm of matrices*, Linear Multilinear Algebra. **60** (2012) 27–31.

Paper JMI-06-59