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A NOTE ON A CERTAIN BIVARIATE MEAN

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Abstract. Weighted arithmetic or geometric means of two bivariate means are used to obtain lower and upper bounds for a bivariate mean introduced by Neuman and Sándor. Bounds involving weighted arithmetic means are sharp.

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