

## A SIMPLE PROOF OF OPPENHEIM'S DOUBLE INEQUALITY RELATING TO THE COSINE AND SINE FUNCTIONS

FENG QI, QIU-MING LUO AND BAI-NI GUO

*Abstract.* In the paper, the authors provide a simple proof of Oppenheim's double inequality relating to the cosine and sine functions. In passing, the authors survey this topic.

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