

## WILKER AND HUYGENS TYPE INEQUALITIES FOR THE LEMNISCATE FUNCTIONS

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*Abstract.* In this paper, we establish Wilker and Huygens type inequalities for the Lemniscate functions.

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