

## AN INEQUALITY ON JORDAN–VON NEUMANN CONSTANT AND JAMES CONSTANT ON $Z_{p,q}$ SPACE

CHANGSEN YANG AND HAIYING LI

*Abstract.* Let  $\lambda > 0$ ,  $Z_{p,q}$  denote  $\mathbb{R}^2$  endowed with the norm

$$|x|_{p,q} = (\|x\|_p^2 + \lambda \|x\|_q^2)^{\frac{1}{2}}.$$

Recently, James constant  $J(Z_{p,q})$  and von Neumann-Jordan constant  $C_{NJ}(Z_{p,q})$  have been investigated under the two cases of a space  $2 \leq p \leq q \leq \infty$  and  $1 \leq p \leq q \leq 2$ . In this note, we show an inequality on these two constants under the case of  $1 \leq p \leq 2 \leq q \leq \infty$ . As an application, we give a sufficient condition for the space  $Z_{p,q}$  with uniform normal structure.

*Mathematics subject classification (2010):* 46B20.

*Keywords and phrases:* Jordan-von Neumann constant, James constant, uniform normal structure.

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