Journal of Mathematical Inequalities
Volume 7, Number 1 (2013), 129–137
doi:10.7153/jmi-07-12

SOME FIXED POINT PROPERTY FOR MULTIVALUED NONEXPANSIVE MAPPINGS IN BANACH SPACES

ZHANFEI ZUO

Abstract. We show some geometric conditions on a Banach space X concerning the generalized James constant, the generalized Jordan-von Neumann constant, the generalized Zbagănu constant and the coefficient $R(1,X)$, which imply the existence of fixed points for multivalued nonexpansive mappings. Our results extend and improve some recent results.

Mathematics subject classification (2010): Primary 47H10, Secondary 46B20.
Keywords and phrases: Multivalued nonexpansive mapping, fixed point, generalized James constant, generalized Jordan-von Neumann constant, generalized Zbagănu constant.

REFERENCES

[1] S. B. NADLER JR., *Multivalued contraction mappings*, Pacific J. Math. **30** (1969), 475–488.
[2] W. A. KIRK, *A fixed point theorem for mappings which do not increase distances*, Amer. Math. Monthly **72** (1965), 1004–1006.
[3] S. DHOMPONGSA, A. KAEWCHAROEN, A. KAEWKHAO, *The Domínguez-Lorenzo condition and multivalued nonexpansive mappings*, Nonlinear Anal. **64** (2006), 958–970.
[4] S. DHOMPONGSA, T. DOMÍNGUEZ BENAVIDES, A. KAEWCHAROEN, A. KAEWKHAO, B. PANYANAK, *The Jordan-von Neumann constant and fixed points for multivalued nonexpansive mappings*, J. Math. Anal. Appl. **320** (2006), 916–927.
[5] A. KAEWKHAO, *The James constant, the Jordan-von Neumann constant weak orthogonality and fixed points for multivalued mappings*, Journal of Mathematical Analysis and Applications **333**, 2 (2007), 950–958.
[6] B. GAVIRA, *Some geometric conditions which imply the fixed point property for multivalued nonexpansive mappings*, Journal of Mathematical Analysis and Applications **339**, 1 (2008), 680–690.
[7] T. DOMÍNGUEZ BENAVIDES AND B. GAVIRA, *The fixed point property for multivalued nonexpansive mappings*, Journal of Mathematical Analysis and Applications **328**, 2 (2007), 1471–1483.
[8] T. DOMÍNGUEZ BENAVIDES AND B. GAVIRA, *Does Kirk's theorem hold for multivalued nonexpansive mappings?*, Fixed Point Theory and Applications Vol. **2010** (2010), Article ID 546761, 20 pages.
[9] J. GAO AND K.-S. LAU, *On two classes of Banach spaces with uniform normal structure*, Studia Math. **99**, 1 (1991), 41–56.
[10] A. JIMÉNEZ-MELADO, E. LLORENS-FUSTER, AND S. SAEJUNG, *The von Neumann-Jordan constant weak orthogonality and normal structure in Banach spaces*, Proc. Amer. Math. Soc. **134** (2006), 355–364.
[11] S. SAEJUNG, *On James and von Neumann-Jordan constants and sufficient conditions for the fixed point property*, J. Math. Anal. Appl. **323** (2006), 1018–1024.
[12] ZHANFEI ZUO, YUNAN CUI, *On some parameters and the fixed point property for multivalued nonexpansive mapping*, Journal of Mathematical Science: Advances and Applications **1** (2008), 183–199.
[13] ZHANFEI ZUO, YUNAN CUI, *A note on the modulus of U-convexity and modulus of W*-convexity*, Journal of Inequalities in Pure and Applied Mathematics **9**, 4 (2008), 1–7.
[14] ZHANFEI ZUO, YUNAN CUI, *Some modulus and normal structure in Banach space*, Journal of Inequalities and Applications Vol. **2009** (2009), Article ID 676373.
[15] ZHANFEI ZUO, YUNAN CUI, *A coefficient related to some geometrical properties of Banach space*, Journal of Inequalities and Applications, Vol. **2009** (2009), Article ID 934321.

Paper JMI-07-12

[16] ZHANFEI ZUO, YUNAN CUI, *The application of generalization modulus of convexity in fixed point theory*, Journal of Natural Science of Heilongjiang University **2** (2009), 206–210.
[17] ZHANFEI ZUO, YUNAN CUI, *Some sufficient conditions for fixed points of multivalued nonexpansive mappings*, Fixed Point Theory and Applications, Vol. **2009** (2009), Article ID 319804.
[18] J. A. CLARKSON, *Uniformly convex spaces*, Transactions of the American Mathematical Society **40**, 3 (1936), 396–414.
[19] B. SIMS, *Orthogonality and fixed points of nonexpansive maps*, Proc. Centre Math. Anal. Austral. Nat. Univ., Canberra **20** (1988), 178–186.
[20] T. DOMÍNGUEZ BENAVIDES, *A geometrical coefficient implying the fixed point property and stability results*, Houston Journal of Mathematics **22** (1996), 835–849.
[21] E. M. MAZCUÑÁN-NAVARRO, *Banach spaces properties sufficient for normal structure*, J. Math. Anal. Appl. **337** (2008), 197–218.