OPERATOR INEQUALITIES RELATED TO WEAK 2–POSITIVITY

MOHAMMAD SAL MOSLEHIAN AND JUN ICHI FUJII

Abstract. In this paper we introduce the notion of weak 2-positivity and present some examples. We establish some operator Cauchy–Schwarz inequalities involving the geometric mean and give some applications. In particular, we present some operator versions of Hua's inequality by using the Choi–Davis–Jensen inequality.

Mathematics subject classification (2010): Primary 47A63; secondary 46L05, 47A30.
Keywords and phrases: Operator inequality, weakly 2-positive map, operator geometric mean, Hua's inequality, Cauchy–Schwarz inequality.

REFERENCES

[1] T. ANDO, *Topics on operator inequality*, Hokkaido Univ. Lecture Note, 1978.
[2] LJ. ARAMBAŠIĆ, D. BAKIĆ AND M. S. MOSLEHIAN, *A treatment of the Cauchy–Schwarz inequality in C*-modules*, J. Math. Anal. Appl. **381** (2011) 546–556.
[3] M.-D. CHOI, *A Schwartz inequality for positive linear maps on C*-algebras*, Illinois J. Math. **18** (1974), 565–574.
[4] M.-D. CHOI, *Some assorted inequalities for positive linear maps on C*-algebras*, J. Operator Theory **4** (1980), 271–285.
[5] C. DAVIS, *A Schwartz inequality for convex operator functions*, Proc. Amer. Math. Soc. **8** (1957), 42–44.
[6] S. S. DRAGOMIR, *A survey on Cauchy–Bunyakovsky–Schwarz type discrete inequalities*, J. Inequal. Pure Appl. Math., **4** (2003), no. 3, Article 63, 142 pp.
[7] R. DRNOVŠEK, *An operator generalization of the Lo-Keng Hua inequality*, J. Math. Anal. Appl., **196** (1995), no. 3, 1135–1138.
[8] J. I. FUJII, *Operator inequalities for Schwarz and Hua*, Sci. Math. **2** (1999), no. 3, 263–268.
[9] J. I. FUJII, *Operator-valued inner product and operator inequalities*, Banach J. Math. Anal. **2** (2008), no. 2, 59–67.
[10] M. FUJII, S. IZUMINO, R. NAKAMOTO AND Y. SEO, *Operator inequalities related to Cauchy–Schwarz and Holder–McCarthy inequalities*, Nihonkai Math. J. **8** (1997) 117–122.
[11] T. FURUTA, J. MIĆIĆ HOT, J. PEČARIĆ AND Y. SEO, *Mond–Pečarić Method in Operator Inequalities*, Element, Zagreb, 2005.
[12] F. HANSEN AND G. K. PEDERSEN, *Jensen's operator inequality*, Bull. London Math. Soc. **35** (2003), no. 4, 553–564.
[13] D. ILIŠEVIĆ AND S. VAROŠANEC, *On the Cauchy–Schwarz inequality and its reverse in semi-inner product C*-modules*, Banach J. Math. Anal. **1** (2007), 78–84.
[14] D. R. JOCIĆ, *Cauchy–Schwarz and means inequalities for elementary operators into norm ideals*, Proc. Amer. Math. Soc. **126** (1998), no. 9, 2705–2711.
[15] M. JOIŢA, *On the Cauchy–Schwarz inequality in C*-algebras*, Math. Rep. (Bucur.) **3** (53) (2001), no. 3, 243–246.
[16] F. KITTANEH, *Some norm inequalities for operators*, Canad. Math. Bull. **42** (1999), no. 1, 87–96.
[17] M. S. MOSLEHIAN, *Operator extensions of Hua's inequality*, Linear Algebra Appl. **430** (2009), no. 4, 1131–1139.
[18] M. S. MOSLEHIAN AND L.-E. PERSSON, *Reverse Cauchy–Schwarz inequalities for positive C*-valued sesquilinear forms*, Math. Inequal. Appl. **12** (2009), no. 4, 701–709.

[19] S. Radas and T. Šikić, *A note on the generalization of Hua's inequality*, Tamkang J. Math., **28** (1997), no. 4, 321–323.

[20] K. Tanahashi, A. Uchiyama and M. Uchiyama, *On Schwarz type inequalities*, Proc. Amer. Math. Soc. **131** (2003), no. 8, 2549–2552.

[21] S. Wada, *On some refinement of the Cauchy–Schwarz inequality*, Linear Algebra Appl. **420** (2007), no. 2–3, 433–440.

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