

INEQUALITIES OF J-P-S-F TYPE

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Abstract. By means of the theory of majorization and under the proper hypotheses, the following inequalities of Jensen-Pečarić-Svrtnan-Fan (Abbreviated as J-P-S-F) type are established:

$$\frac{f(A(x))}{g(A(x))} \leq \dots \leq \frac{f_{k+1,n}(x)}{g_{k+1,n}(x)} \leq \frac{f_{k,n}(x)}{g_{k,n}(x)} \leq \dots \leq \frac{A(f(x))}{A(g(x))},$$

where

$$f_{k,n}(x) := \frac{1}{\binom{n}{k}} \sum_{1 \leq i_1 < \dots < i_k \leq n} f\left(\frac{x_{i_1} + \dots + x_{i_k}}{k}\right), \quad x \in [a, b]^n.$$

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