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APPROXIMATION OF CONJUGATE FUNCTIONS BY TRIGONOMETRIC POLYNOMIALS IN WEIGHTED ORLICZ SPACES

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Abstract. We investigate the approximation of a conjugate function by the Fejér sums of the Fourier series of the conjugate function and obtain the estimate between the derivatives of the conjugate functions and the derivatives of the conjugate trigonometric polynomials in the weighted Orlicz spaces with Muckenhoupt weights. We prove inverse theorem of approximation theory for the derivatives conjugate functions in the weighted Orlicz spaces.

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