n−EXPONENTIAL CONVEXITY FOR JENSEN−TYPE INEQUALITIES

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Abstract. Starting from the results given in [14] where the uniform treatment of the Jensen type inequalities and its converses is given, we investigate the exponential convexity of differences of the left-hand and the right-hand side of these inequalities. Using these differences, we produce new exponentially convex functions. Finally, we give several examples of the families of functions for which the obtained results can be applied, and we get some generalized Cauchy means. Results from this paper present the generalization of the results from [2].

Mathematics subject classification (2010): 26D15.

Keywords and phrases: Green function, Jensen inequality, converse of the Jensen inequality, Cauchy-type mean value theorems, $n-$ exponential convexity, exponential convexity, log-convexity, means.

REFERENCES

[1] M. ANWAR, J. JAKŠETIĆ, J. PEČARIĆ, ATIQ UR REHMAN, *Exponential convexity, positive semi-definite matrices and fundamental inequalities*, J. Math. Inequal. **4**, 2 (2010), 171–189.

[2] M. ANWAR, J. PEČARIĆ, *Cauchy's means for signed measures*, Bull. Malays. Math. Sci. Soc. (2) **34**, 1 (2011), 31–38.

[3] R. P. BOAS, *The Jensen-Steffensen inequality*, Publ. Elektroteh. Fak., Univ. Beogr., Ser. Mat. **302–319** (1970), 1–8.

[4] H. D. BRUNK, *On an inequality for convex functions*, Proc. Amer. Math. Soc. **7** (1956), 817–824.

[5] P. S. BULLEN, *Means and Their Inequalities*, Kluwer Academic Publishers, Dordrecht/Boston/London, 2003.

[6] J. JAKŠETIĆ, J. PEČARIĆ, *Exponential Convexity Method*, J. Convex Anal., to appear.

[7] M. KLARIČIĆ BAKULA, J. PEČARIĆ, J. PERIĆ, *Extensions of the Hermite-Hadamard inequality with applications*, Math. Inequal. Appl. **12** (2012), 899–921.

[8] A. MCD. MERCER, *An "error term" for the Ky Fan inequality*, J. Math. Anal. Appl. **220** (1998), 774–777.

[9] A. MCD. MERCER, *Some new inequalities involving elementary mean values*, J. Math. Anal. Appl. **229** (1999), 677–681.

[10] J. E. PEČARIĆ, *A new proof of the Jensen-Steffensen inequality*, Mathematica **23**, 46 (1981), 73–77.

[11] J. E. PEČARIĆ, *On variants of Jensen's inequality*, Glas. Mat., III. Ser. **18**, 38 (1983), 281–289.

[12] J. E. PEČARIĆ, *Konveksne funkcije: Nejednakosti*, Naučna knjiga, Beograd, 1987.

[13] J. E. PEČARIĆ, I. PERIĆ, H. M. SRIVASTAVA, *A family of the Cauchy type mean-value theorems*, J. Math. Anal. Appl. **306** (2005), 730–739.

[14] J. PEČARIĆ, I. PERIĆ, M. RODIĆ LIPANOVIĆ, *Uniform treatment of Jensen type inequalities*, to appear in Math. Rep. (Bucur.)

[15] J. E. PEČARIĆ, F. PROSCHAN, Y. L. TONG, *Convex functions, partial orderings, and statistical applications*, Mathematics in science and engineering, Vol. **187**, Academic Press, 1992.

[16] D. V. WIDDER, *Completely convex function and Lidstone series*, Trans. Am. Math. Soc. **51** (1942), 387–398.