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THE WEIGHTED ESTIMATE FOR THE COMMUTATOR OF THE GENERALIZED FRACTIONAL INTEGRAL

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Abstract. Let L be the infinitesimal generator of an analytic semigroup on $L^2(\mathbb{R}^n)$ with Gaussian kernel bound, and let $L^{-\alpha/2}$ be the fractional integral of L for $0<\alpha<n$. Suppose that b is a locally integral function, then the commutator generated by b and $L^{-\alpha/2}$ is defined by $[b,L^{-\alpha/2}](f)=bL^{-\alpha/2}(f)-L^{-\alpha/2}(bf)$. When b belongs to weighted Lipschitz function space, the boundedness of $[b,L^{-\alpha/2}]$ from $L^p(\omega,\mathbb{R}^n)$ to $L^q(\omega^{1-(1-\alpha/n)q},\mathbb{R}^n)$ is established, where $1<p<\infty$, $0<\beta<1$ and $1/q=1/p-(\alpha+\beta)/n$ with $1/p>(\alpha+\beta)/n$.

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