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SHARP TWO PARAMETER BOUNDS FOR THE LOGARITHMIC MEAN AND THE ARITHMETIC–GEOMETRIC MEAN OF GAUSS

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Abstract. For fixed $s \geqslant 1$ and $t_1, t_2 \in (0,1/2)$ we prove that the inequalities $G^s(t_1a+(1-t_1)b, t_1b+(1-t_1)a)A^{1-s}(a,b) > AG(a,b)$ and $G^s(t_2a+(1-t_2)b, t_2b+(1-t_2)a)A^{1-s}(a,b) > L(a,b)$ hold for all $a,b>0$ with $a \neq b$ if and only if $t_1 \geqslant 1/2-\sqrt{2s}/(4s)$ and $t_2 \geqslant 1/2-\sqrt{6s}/(6s)$. Here $G(a,b)$, $L(a,b)$, $A(a,b)$ and $AG(a,b)$ are the geometric, logarithmic, arithmetic and arithmetic-geometric means of a and b, respectively.

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