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DIFFERENCE OF COMPOSITION OPERATORS ON HARDY SPACE

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Abstract. Suppose φ is an analytic self-map of open unit disk $\mathbb{D}$ and w is an analytic function on $\mathbb{D}$. Then a weighted composition operator induced by φ with weight w is given by $(W_{w,\varphi}f)(z) = w(z)f(\varphi(z))$, for $z \in \mathbb{D}$ and f analytic on $\mathbb{D}$. We find a sufficient condition under which two composition operators lie in the same path component of $\mathscr{C}(H^2)$, and we find a sufficient condition for the difference of such operators to be compact on $H^2(\mathbb{D})$. Then we provide another example that answers a question raised by Shapiro and Sundberg [18] negatively. Moreover, we characterize the Hilbert-Schmidt difference of two composition operators on $H^2(\mathbb{D})$.

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