

THE EQUIVALENCE OF CONVERGENCE THEOREMS OF ISHIKAWA–MANN ITERATIONS WITH ERRORS FOR Φ -CONTRACTIVE MAPPINGS IN UNIFORMLY SMOOTH BANACH SPACES

ZHIQUN XUE

Abstract. Let E be an arbitrary uniformly smooth real Banach space, D be a nonempty closed convex subset of E , and $T : D \rightarrow D$ a generalized Lipschitz Φ -contractive mapping with $q \in F(T) \neq \emptyset$. Let $\{a_n\}, \{b_n\}, \{c_n\}, \{d_n\}$ be four real sequences in $[0, 1]$ and satisfy the conditions: (i) $a_n + c_n \leq 1$, $b_n + d_n \leq 1$; (ii) $a_n, b_n, d_n \rightarrow 0$ as $n \rightarrow \infty$; (iii) $c_n = o(a_n)$; (iv) $\sum_{n=0}^{\infty} a_n = \infty$. For some $x_0, z_0 \in D$, let $\{u_n\}, \{v_n\}, \{w_n\}$ be any bounded sequences in D , and $\{x_n\}$ and $\{z_n\}$ be Ishikawa and Mann iterative sequences with errors defined by (1.1) and (1.2), respectively. Then the convergence of (1.1) is equivalent to that of (1.2).

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