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ON A KY FAN TYPE INEQUALITY DUE TO H. ALZER

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Abstract. Let A_n and H_n (respectively, A'_n and H'_n) be the weighted arithmetic and harmonic means of $x_1, x_2, \ldots, x_n$ (respectively, $1-x_1, 1-x_2, \ldots, 1-x_n$), where $x_i \in (0, 1/2]$ $(i = 1, 2, \ldots, n;\ n \geqslant 2)$. We mainly show that, if not all of the x_i's are equal, then

$$\min_{1\leqslant i\leqslant n} \frac{x_i}{1-x_i} < \frac{A'_n - H'_n}{A_n - H_n} < \max_{1\leqslant i\leqslant n} \frac{x_i}{1-x_i},$$

which is a refinement and converse of the Ky Fan type inequality $A'_n - H'_n \leqslant A_n - H_n$ due to H. Alzer. Some parallel and related results are also discussed.

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