ON A KY FAN TYPE INEQUALITY DUE TO H. ALZER

JAMAL ROOIN

Abstract. Let $A_n$ and $H_n$ (respectively, $A'_n$ and $H'_n$) be the weighted arithmetic and harmonic means of $x_1, x_2, \ldots, x_n$ (respectively, $1 - x_1, 1 - x_2, \ldots, 1 - x_n$), where $x_i \in (0, 1/2]$ ($i = 1, 2, \ldots, n; n \geq 2$). We mainly show that, if not all of the $x_i$’s are equal, then

$$\min_{1 \leq i \leq n} \frac{x_i}{1 - x_i} < \frac{A'_n - H'_n}{A_n - H_n} < \max_{1 \leq i \leq n} \frac{x_i}{1 - x_i},$$

which is a refinement and converse of the Ky Fan type inequality $A'_n - H'_n \leq A_n - H_n$ due to H. Alzer. Some parallel and related results are also discussed.


Keywords and phrases: A-G-H inequality, Ky Fan’s inequality.

REFERENCES