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SIMPLE PROOFS OF THE CUSA–HUYGENS–TYPE AND BECKER–STARK–TYPE INEQUALITIES

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Abstract. In this paper, we respectively give some simple proofs of the Cusa-Huygens- and Becker-Stark-type inequalities presented by Chen and Cheung in [9].

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