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OPTIMAL INEQUALITIES BETWEEN NEUMAN–SÁNDOR, CENTROIDAL AND HARMONIC MEANS

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Abstract. In this paper, we answer the question: what are the greatest values α_1, α_2 and the least values β_1, β_2, such that the inequalities

$$\alpha_1 T(a,b) + (1-\alpha_1)H(a,b) < R(a,b) < \beta_1 T(a,b) + (1-\beta_1)H(a,b)$$

and

$$T^{\alpha_2}(a,b)H^{1-\alpha_2}(a,b) < R(a,b) < T^{\beta_2}(a,b)H^{1-\beta_2}(a,b)$$

hold for all $a,b > 0$ with $a \neq b$? Here, $R(a,b)$, $T(a,b)$ and $H(a,b)$ denote the Neuman-Sándor, centroidal and harmonic means of two positive numbers a and b, respectively.

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