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REFINEMENTS OF BOUNDS FOR THE FIRST AND SECOND SEIFFERT MEANS

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Abstract. In this paper, we find the greatest values α, λ and the least values β, μ such that the double inequalities $\alpha[5A(a,b)/6+H(a,b)/6]+(1-\alpha)A^{5/6}(a,b)H^{1/6}(a,b) < P(a,b) < \beta[5A(a,b)/6+H(a,b)/6]+(1-\beta)A^{5/6}(a,b)H^{1/6}(a,b)$ and $\lambda\,[A(a,b)/3+2Q(a,b)/3]+(1-\lambda)A^{1/3}(a,b)Q^{2/3}(a,b) < T(a,b) < \mu\,[A(a,b)/3+2Q(a,b)/3]+(1-\mu)A^{1/3}(a,b)Q^{2/3}(a,b)$ hold for all $a,b>0$ with $a\neq b$. Here $A(a,b)$, $H(a,b)$, $Q(a,b)$, $P(a,b)$ and $T(a,b)$ denote the arithmetic, harmonic, quadratic, first Seiffert and second Seiffert means of two positive numbers a and b, respectively.

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