

THE APPLICATIONS ON SOME INEQUALITIES OF THE COMPOSITION OF ENTIRE FUNCTIONS

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Abstract. The purpose of our paper is to deal with some growth problem of two composite entire functions of finite $[p, q]$ -order and some properties of growth of $f \circ g$, f and g . Some results are obtained as follows: Let f, g be two entire functions and have index-pair $[p_1, q_1]$, $[p_2, q_2]$, respectively. Let $f \circ g$ have index-pair $[p_3, q_3]$. Then we have the following conclusions:

(i) if $p_2 + 1 - q_1 > 0$, then $p_3 = p_1 + p_2 - q_1 + 1 \iff q_3 = q_2$;

(ii) if $p_2 + 1 - q_2 = 0$, then $p_3 = p_1 \iff q_3 = q_2$;

(iii) if $p_2 + 1 - q_1 < 0$, then $p_3 = q_1 \iff q_3 = q_1 + q_2 - p_2 - 1$.

These results are some improvement and generalization of the form theorems given by Gross, Lahiri, Tu.

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