

INEQUALITY CHAINS FOR WILKER, HUYGENS AND LAZAREVIĆ TYPE INEQUALITIES

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Abstract. We offer various refinements of inequalities related to the Wilker, Huygens, or Lazarević type inequalities for trigonometric and hyperbolic functions.

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