ON THE BOUNDEDNESS OF MAXIMAL AND POTENTIAL OPERATORS IN VARIABLE EXPONENT AMALGAM SPACES

ALEXANDER MESKHI AND MUHAMMAD ASAD ZAIGHUM

Abstract. Two–weight estimates for maximal and fractional integral operators in variable exponent amalgam spaces $(L^{p(\cdot)}, l^q)$ are established under the log– Hölder continuity condition on the exponent $p(\cdot)$. Some of the derived results are new even for constant p.

Mathematics subject classification (2010): 42B25, 46E30.

Keywords and phrases: Variable exponent amalgam spaces, maximal operator, potentials, boundedness.

REFERENCES

[1] R. ABOULAICH, D. MESKINE AND A. SOUISSI, *New diffusion models in image processing*, Comput. Math. Appl. **56**, 4 (2008), 874–882.

[2] D. ADAMS, *A trace inequality for generalized potentials*, Studia Math. **48** (1973), 99–105.

[3] M.I. AGUILAR CAÑESTRO AND P. ORTEGA SALVADOR, *Boundedness of generalized Hardy operators on weighted amalgam spaces*, Math. Inequal. Appl. **13**, 2 (2010), 305–318.

[4] M.I. AGUILAR CAÑESTRO AND P. ORTEGA SALVADOR, *Boundedness of positive operators on weighted amalgams*, J. Inequal. Appl. **2011**, 13 (2011), DOI:10.1186/1029-242X-2011-13.

[5] G. BENNETT AND R. SHARPLY, *Interpolation of operators*, Pure and Appl. Math. **129**, Academic press, 1988.

[6] C. CAPONE, D. CRUZ-URIBE SFO AND A. FIORENZA, *The fractional maximal operator on variable L^p spaces*, Revista Mat. Iberoamericana **3**, 23 (2007), 747–770.

[7] C. CARTON-LEBRUN, H. P. HEINIG AND S. C. HOFMANN, *Integral operators on weighted amalgams*, Studia Math. **2** (1994), 133–175.

[8] D. CRUZ-URIBE, L. DIENING AND P. HÄSTÖ, *The maximal operator on weighted variable Lebesgue spaces*, Frac. Calc. Appl. Anal. **14**, 3 (2011), 361–374.

[9] D. CRUZ-URIBE SFO, A. FIORENZA, J. M. MARTELL AND C. PEREZ, *The boundedness of classical operators on variable L^p spaces*, Ann. Acad. Sci. Fenn. Math. **31** (2006), 239–264.

[10] M. M. DAY, *Some more uniformly convex spaces*, Bull. Amer. Math. Soc. **47** (1941), 504–507.

[11] L. DIENING, *Maximal function on generalized Lebesgue spaces $L^{p(\cdot)}$*, Math. Inequal. Appl. **7**, 2 (2004), 245–253.

[12] L. DIENING, *Riesz potentials and Sobolev embeddings on generalized Lebesgue and Sobolev spaces $L^{p(\cdot)}$ and $W^{k,p(\cdot)}$*, Math. Nachr. **268** (2004), 31–34.

[13] L. DIENING, P. HARJULEHTO, P. HÄSTÖ AND M. RŮŽIČKA, *Lebesgue and Sobolev spaces with variable exponents*, Lecture Notes in Mathematics, vol. **2017**, Springer-Verlag, Berlin, 2011.

[14] L. DIENING AND P. HÄSTÖ, *Muckenhoupt weights in variable exponent spaces*, Preprint, Available at http://www.helsinki.fi/ pharjule/varsob/publications.shtml.

[15] D. E. EDMUNDS AND A. MESKHI, *Potential-type operators in $L^{p(x)}$ spaces*, Z. Anal. Anwendungen **21** (2002), 681–690.

[16] J. F. FOURNIER AND S. WATSON, *Amalgams of L^p and l^q*, Bull. Amer. Math. Soc. (N.S.). **13**, 1 (1985), 1–21.

[17] P. HARJULEHTO, P. HÄSTÖ, AND M. PERE, *Variable exponent Lebesgue spaces on metric spaces: the Hardy-Littlewood maximal operator*, Real Anal. Exchange **30** (2004-2005), 87–104.

[18] H. P. HEINIG AND A. KUFNER, *Weighted Friedrichs inequalities in amalgams*, Czechoslovak Math. J. **43(118)**, 2 (1993), 285–308.

[19] G. KÖTHE, *Topological vector spaces I*, Springer-Verlag, 1969.

[20] V. KOKILASHVILI AND A. MESKHI, *Weighted criteria for generalized fractional maximal functions and potentials in Lebesgue spaces with variable exponent*, Integr. Trans. Spec. Func. **18**, 9 (2007), 609–628.

[21] V. KOKILASHVILI AND A. MESKHI, *Two–weight inequalities for fractional maximal functions and singular integrals in $L^{p(\cdot)}$ spaces*, J. Math. Sci., Springer **173**, 6 (2011), 656–673.

[22] V. KOKILASHVILI, A. MESKHI AND M. SARWAR, *One and two weight estimates for one-sided operators in $L^{p(\cdot)}$ spaces*, Eurasian Math. J. **1**, 1 (2010), 73–110.

[23] V. KOKILASHVILI, A. MESKHI AND M. A. ZAIGHUM, *Weighted kernel operators in variable exponent amalgam spaces*, J. Inequal. Appl. **2013**, 173 (2013), DOI:10.1186/1029-242X-2013-173.

[24] V. KOKILASHVILI AND S. SAMKO, *Maximal and fractional operators in weighted $L^{p(x)}$ spaces*, Rev. Mat. Iberoamericana **20**, 2 (2004), 493–515.

[25] V. KOKILASHVILI AND S. SAMKO, *On Sobolev theorem for Riesz-type potentials in Lebesgue spaces with variable exponent*, Z. Anal. Anwendungen **22**, 4 (2003), 899–910.

[26] O. KOVÁCIK AND J. RÁKOSNÍK, *On spaces $L^{p(x)}$ and $W^{k,p(x)}$*, Czechoslovak Math. J. **41**(116), 4 (1991), 592–618.

[27] V. G. MAZ'YA AND I. E. VERBITSKY, *Capacitary inequalities for fractional integrals, with applications to partial differential equations and Sobolev multipliers*, Ark. Mat. **33** (1995), 81–115.

[28] F. MAMEDOV AND Y. ZEREN, *On a two–weighted estimation of maximal operator in the Lebesgue space with variable exponent*, Annali di Matematica, Doi 10.1007/s10231-010-0149.

[29] A. MESKHI, *Measure of Non-Compactness for Integral Operators in Weighted Lebesgue Spaces*, Nova Science Publishers, New York, 2009.

[30] P. ORTEGA SALVADOR AND C. RAMÍREZ TORREBLANCA, *Hardy operators on weighted amalgams*, Proc. Roy. Soc. Edinburgh Sect. A **140**, 1 (2010), 175–188.

[31] Y. RAKOTONDRATSIMBA, *Fractional maximal and integral operators on weighted amalgam spaces*, J. Korean Math. Soc. **36**, 5 (1999), 855–890.

[32] M. RUŽIČKA, *Electrorheological fluids: modeling and mathematical theory*, Lecture Notes in Mathematics Vol. **1748**, Springer-Verlag, Berlin, 2000.

[33] S. SAMKO, *Convolution type operators in $L^{p(x)}$*, Integral Transforms Spec. Funct. **7**, 1–2 (1998), 123–144.

[34] S. SAMKO, *Convolution type operators in $L^{p(x)}(R^n)$*, Integral Transforms Spec. Funct. **7**, 3–4 (1998), 261–284.

[35] E. SAWYER, *Two–weight norm inequalities for certain maximal and integral operators*, in: Harmonic analysis, Minneapolis, Minn. 1981, pp. 102–127, Lecture Notes in Math. Vol. 908, Springer, Berlin, New York, 1982.

[36] E. SAWYER AND R. L. WHEEDEN, *Weighted inequalities for fractional integrals on Euclidean and homogeneous Spaces*, Amer. J. Math. **114**, 4, 813–874.

[37] I. I. SHARAPUDINOV, *The topology of the space $\mathscr{L}^{p(t)}([0,1])$* (Russian), Mat. Zametki **26**, 4 (1979), 613–632.

[38] J. STEWART AND S. WATSON, *Irregular Amalgams*, Internat. J. Math. Math. Sci. **9**, 2 (1986), 331–340.

[39] I. E. VERBITSKY, *Weighted norm inequalities for maximal operators and Pisier's Theorem on factorization through $L^{p,\infty}$*, Integr. Equ. Oper. Theory **15** (1992), 124–153.

[40] N. WIENER, *On the representation of functions by trigonometrical integrals*, Math.Z. **24** (1926), 575–616.

[41] N. WIENER, *Tauberian theorem*, Ann. of Math. **33** (1932), 1–100.

Journal of Mathematical Inequalities
www.ele-math.com
jmi@ele-math.com