

## ON SOME MEANS DERIVED FROM THE SCHWAB–BORCHARDT MEAN

EDWARD NEUMAN

*Abstract.* Bivariate means defined as the Schwab-Borchardt mean of two bivariate means are investigated. Explicit formulas for those means are obtained. It is demonstrated that they interpolate inequalities connecting the well known bivariate means. Optimal bounds for the means under discussion are also obtained. The bounding quantities are convex combinations of the generating means.

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