

## CONVERSES OF A DISCRETE WIRTINGER TYPE INEQUALITY

LENG TUO AND FENG YONG

**Abstract.** In this paper, we establish the following converse of Alzer's inequality, which is a discrete analogue of Wirtinger's inequality: Let  $z_1, z_2, \dots, z_n (n \geq 2)$  be complex numbers with

$$\sum_{k=1}^n z_k = 0,$$

then

$$\sum_{k=1}^n |z_k|^2 \geq \lambda(n) \min_{1 \leq k \leq n} \{|z_{k+1} - z_k|^2\}$$

where  $z_{n+1} = z_1$  and  $\lambda(n) = \frac{n}{4}$ , for even  $n$ ;  $\lambda(n) = \frac{n}{4 \sin^2 \frac{(n-1)\pi}{2n}}$ , for odd  $n$ . The constant  $\lambda(n)$  is best possible.

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