

LOCAL HÖLDER ESTIMATES FOR GENERAL ELLIPTIC $p(x)$ -LAPLACIAN EQUATIONS

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Abstract. In this paper we obtain the interior Hölder regularity of the gradients of weak solutions for general elliptic $p(x)$ -Laplacian equations

$$\operatorname{div}(a(x, \nabla u)) = \operatorname{div}\left(|\mathbf{f}|^{p(x)-2}\mathbf{f}\right),$$

under some proper assumptions on a and the Hölder continuous functions p, \mathbf{f} .

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